A Graphical Overview of Tri-Criterion Portfolio Selection

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Fifty years ago, Markowitz introduced the risk-return efficient frontier and an algorithm called the critical line method for computing it. The idea is to present the set of all contenders for optimality and then have the decision maker select from its graph his or her most preferred portfolio. Being able to view the entire efficient set at once is desirable because a decision maker can readily see why a most preferred solution is best – as all of the other contenders for optimality are seen to be less satisfactory.

Despite the fact that the approach is at the heart of modern portfolio theory, it has been difficult to extend the methodology to include even one extra criterion, which is the purpose here. This is because the efficient frontier becomes a surface and because the critical line algorithm does not scale to additional criteria. In this paper, a multi-parametric quadratic programming procedure is reviewed for computing the efficient surface of a tri-criterion portfolio selection problem in closed-form (where the extra objective could be dividends, liquidity, turnover, etc.). Actually, the procedure, designed here for portfolio selection, has generality in that it can be used to solve for the nondominated set of any quad-lin-lin multiple objective program, in or out of finance.

Focusing on the efficient surfaces generated in tri-criterion portfolio selection, the paper displays many graphs of the types of efficient surfaces a problem might possess as a function of problem size and other factors.